CHAPTER 2 - GRAPHS

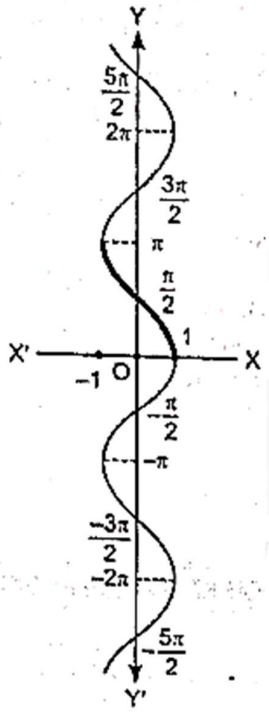
GRAPHS OF DIFFERENT INVERSE TRIGRNOMETRIC RATIO

1) The inverse of cosine function is denoted by $\cos^{-1}$ (or arc cosine) function. Thus

$y = \cos^{-1} x$ if and only if $x = \cos y$

$x \in [-1, 1]$ and $y \in [0, \pi]$

Graph of $y = \cos^{-1} x$

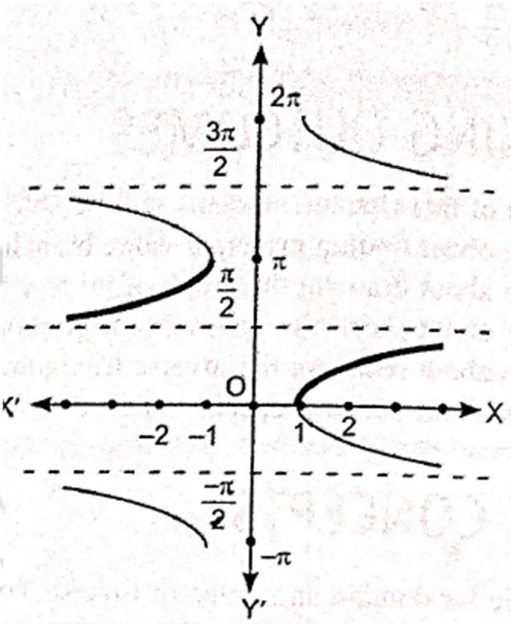


2) The inverse of secant function is denoted by $\sec^{-1}$ (or arc secant) function. Thus

$y = \sec^{-1} x$ if and only if $x = \sec y$

$x \in (-\infty, -1] \cup [1, \infty)$ and $y \in [0, \pi/2) \cup (\pi/2, \pi]$

Graph of $y = \sec^{-1} x$.

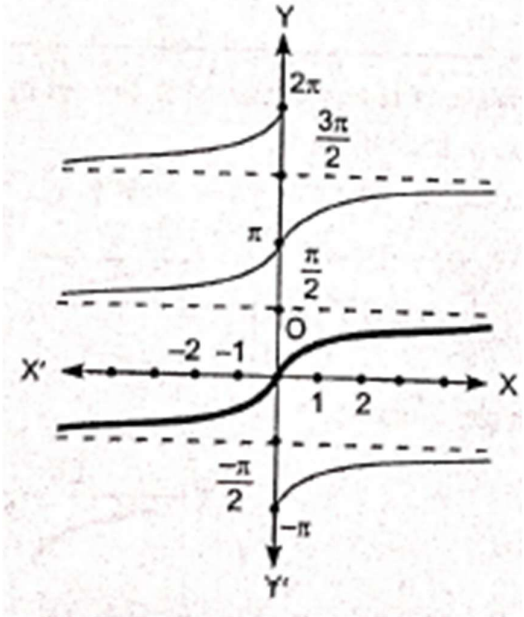


3) The inverse of tangent function is denoted by $\tan^{-1}$ (or arc tangent) function. Thus

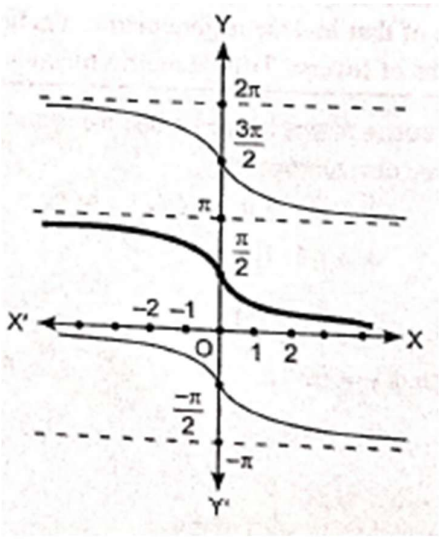
$y = \tan^{-1} x$ if and only if $x = \tan y$

$x \in (-\infty, \infty)$ and $y \in (-\pi/2, \pi/2)$

Graph of $y = \tan^{-1} x$.



4) The inverse of cotangent function is denoted by $\cot^{-1}$ (or arc cotangent) function. Thus
$y = \cot^{-1} x$ if and only if $x = \cot y$
$x \in (-\infty, \infty)$ and $y \in (0, \pi)$
Graph of $y = \cot^{-1} x$.



DAILY PRACTICE QUESTION

✤ **Very Short (Objective Type) / Short Answer Type**

1) Principal value of $\sin^{-1}(-1/2)$ is
(a) $\pi/3$ (b) $-\pi/3$ (c) $5\pi/6$ (d) $-\pi/6$

2) $\sin^{-1}(\sin 2\pi/3) = 2\pi/3$, state true or false.

3) $\tan^{-1}[\sin(-\pi/2)]$ is equal to
(a) -1 (b) 1 (c) $\pi/2$ (d) $-\pi/4$

4) $\sec\{\tan^{-1}(y/3)\}$ is equal to
(a) $\sqrt{(9+y^2)} / 9$ (b) $\sqrt{(9+y^2)} / 3$
(c) $3 / \sqrt{(9+y^2)}$ (d) $9 / \sqrt{(9+y^2)}$

5) Principal branch of $\tan^{-1} x$ is _____

6) Find the value of $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$.

7) Find the principal value of $\cos^{-1}(1/2)$.

8) Find the principal value of $\sin^{-1}(^{-1}/_{2}) + \cos^{-1}(^{-1}/_{2})$.

9) What is the domain of the function $\sin^{-1} x$?

10) Find the value of $\sin[\frac{\pi}{3} - sin^{-1}(-\frac{1}{2})]$.

[NCERT; Delhi 2011]

11) If $cos^{-1}\frac{x}{a} + cos^{-1}\frac{y}{b} = \propto$, prove that $\frac{x^2}{a^2} - \frac{2xy}{ab} + \frac{y^2}{b^2} = sin^2 \propto$.

12) Show that $\tan(1/2 \sin^{-1}(¾)) = (4 - \sqrt{7}) / 3$.

13) Prove that $\sec^2(\tan^{-1}2) + \text{cosec}^2(\cot^{-1}3) = 15$.

14) If $a_1, a_2, a_3, \ldots\ldots, a_n$ is an arithmetic progression with common difference d, then evaluate the following expression

$$tan\left[tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + tan^{-1}\left(\frac{d}{1+a_3a_4}\right) + \cdots + tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right].$$

15) Prove that $2\tan^{-1}(1/5) + \sec^{-1}(5\sqrt{2}/7) + 2\tan^{-1}1/8 = \pi/4$

16) Write the following in the simplest form:
$\tan^{-1}(\cos x / 1 + \sin x)$

17) Simplify: $\tan^{-1}[(a \cos x - b \sin x) / (b \cos x + a \sin x)]$, if $a/b \tan x > -1$.

[NCERT]

18) Solve for x, $\tan^{-1}[(1-x)/(1+x)] = ½ \tan^{-1} x$, $x > 0$.

[NCERT; NCERT Exemplar; Foreign 2011]

19) Solve the following equation:
$\tan^{-1}[(x+1)/(x-1)] + \tan^{-1}[(x-1)/(x-1)] = \tan^{-1}(-7)$.

20) Write the following function in the simplest form :
$\text{Tan}^{-1}[\sqrt{(1+x^2)}-1/x]$, x is not equal to 0.

21) Solve the following equation:
$\text{Tan}^{-1}(x+1)/(x-1) + \tan^{-1}(x-1)/x = \tan^{-1}(-7)$.

22) Find the value of the expression
$\text{Sin}\,(2\tan^{-1}(1/3)) + \cos(\tan^{-1}2\sqrt{2})$

23) Find the value of $\cot \frac{1}{2}[\cos^{-1}(2x/1+x^2)+\sin^{-1}(1-y^2)/(1+y^2)]$, $|x| < 1$, $y > 0$ and $xy < 1$.

24) Prove that: $tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\, cos^{-1}x^2;\ -1 < x < 1$

[Foreign 2017]

11) $\cos^{-1}(\cos 13\pi/6)$. **[NCERT]**

12) Write the principal value of the following: $\sin^{-1}(\sin 4\pi/5)$

13) Using principal value evaluate the following:
$\cos^{-1}(\cos 2\pi/3) + \sin^{-1}(\sin 2\pi/3)$.

[AI 2011]

14) If $\sin^{-1} x + \sin^{-1} y = \pi$, then find the values of x and y.

15) If $x < 0$, $y < 0$, such that $xy = 1$, then find the value of
$\tan^{-1} x + \tan^{-1} y$.

✤ Long Answer I / Long Answer II Type

Prove that in Exercises 16 to 26:

16) $\sin^{-1}(12/13) + \cos^{-1}(4/5) + \tan^{-1}(63/16) = \pi$. **[NCERT]**

17) $\cot^{-1}((\sqrt{1 + \sin x} + \sqrt{1 - \sin x})/(\sqrt{1 + \sin x} - \sqrt{1 - \sin x})) = x/2$, $x \in (0, \pi/4)$

[NCERT; Delhi 2014, 11]

18) $\tan^{-1}(1/4) + \tan^{-1}(2/9) = 1/2\, \cos^{-1}(3/5)$ **[HOTS]**

19) $\cos^{-1}(12/13) + \sin^{-1}(3/5) = \sin^{-1}(56/65)$ **[NCERT; Dehradun 2019]**

20) $\tan^{-1} 2x + \tan^{-1} (4x/(1-4x^2)) = \tan^{-1}((6x-8x^3)/(1-12x^2))$, $|x| < 1/(2\sqrt{3})$ **[Foreign 2017]**

21) $2 \tan^{-1}(1/2) + \tan^{-1}(1/7) = \tan^{-1}(31/17)$. **[NCERT; AI 2011]**

22) $\sin^{-1}(8/17) + \sin^{-1}(3/5) = \cos^{-1}(36/85) = \tan^{-1}(77/36)$ **[NCERT; Delhi 2012, 13 (C)]**

23) $\sin^{-1}(1/\sqrt{5}) + \cot^{-1}(3) = \pi/4$. **[HOTS]**

24) $\cos(\sin^{-1}(3/5) + \sin^{-1}(5/13)) = 33/65$.

25) $\cos^{-1}(4/5) + \cos^{-1}(12/13) = \cos^{-1}(33/65)$. [NCERT]

26) $2 \sin^{-1}(3/5) = \tan^{-1}(24/7)$. [NCERT]

Write the following functions in the simplest form (Exercises 27 and 28):

27) $\tan^{-1} |3x - x^3| / |1 - 3x^2|$

28) $\tan^{-1} \sqrt{(1 - \cos 3x) / (1 + \cos 3x)}$, $x < \pi$

29) Solve for x, $\tan^{-1}(x + 1) + \tan^{-1}(x - 1) = \tan^{-1}(3/31)$. **[Foreign 2015]**

30) Find the value of x, if $\sin [\cot^{-1}(x + 1)] = \cos(\tan^{-1} x)$ **[DoE; Bhubaneswar 2015, Delhi 2015]**

31) Prove the following: $2 \sin^{-1}(3/5) - \tan^{-1}(17/31) = \pi/4$. **[Bhubaneshwar 2015]**

32) Solve the following for x:
$\tan^{-1}((x - 2)/(x - 3)) + \tan^{-1}((x + 2)/(x + 3)) = \pi/4$, $|x| < 1$. **[Patna 2015]**

33) Prove the following:
$\sin [\tan^{-1}((1 - x^2)/2x) + \cos^{-1}((1 - x^2)/(1 + x^2))] = 1$, $0 < x < 1$. **[Guwahati 2015]**

34) Solve for x, $2 \tan^{-1}(\sin x) = \tan^{-1}(2 \sec x)$, $x \neq \pi/2$ **[DoE: Foreign 2012]**

35) Solve for x: $\tan^{-1}(2x / (1 - x^2)) + \cot^{-1}(1 - x^2 / 2x) = 2\pi/3$, $x > 0$.

36) Solve for x: $\cos^{-1}((x^2 - 1) / (x^2 + 1)) + 1/2 \tan^{-1}(2x / (1 - x^2)) = 2x/3$

37) Solve for x: $\sin^{-1}(2\alpha / (1 + \alpha^2)) + \sin^{-1}(2\beta / (1 + \beta^2)) = 2 \tan^{-1}x$. **[HOTS]**

38. Solve for x: $\sin^{-1} x + \sin^{-1} 2x = \pi/3$. [HOTS]

INTEGRATED EXERCISE

✤ Very Short (Objective Type) / Short Answer Type

1. The principal value of $\sin^{-1}(\sin 2\pi/3)$ is
   (a) $2\pi/3$ (b) $\pi/3$ (c) $-\pi/6$ (d) $\pi/6$
2. The value of $\cos^{-1}(1/2) + 3\sin^{-1}(1/2)$ is equal to
   (a) $\pi/4$ (b) $\pi/6$ (c) $2\pi/3$ (d) $5\pi/6$
3. The greatest and least values of $(\sin^{-1} x)^2 + (\cos^{-1} x)^2$ are respectively _____
4. The value of $\sin(2\sin^{-1}(-6))$ is _____
5. Find the principal value of $\text{cosec}^{-1}(2)$. **[NCERT]**
6. Evaluate $\tan^{-1}[\sin(-(\pi/2))]$. **[NCERT Exemplar]**
7. Write the value of $\cos^{-1}(-1/2) + 2\sin^{-1}(1/2)$
   [Foreign 2014]
8. Write one branch of $\sin^{-1} x$ other than the principal branch.
9. Find the principal value of $\tan^{-1}(-1)$
   [NCERT]
10. Find the principal value of $\cos^{-1}(\cos(7\pi/6))$
    [NCERT; HOTS; Delhi 2011]
11. Find the value of $\sin(2\sin^{-1}(3/5))$.
    [Foreign 2013]
12. Find the principal value of $\tan^{-1}(\tan(9\pi/8))$.
    [NCERT Exemplar; Foreign 2013]
13. Write the principal value of $\tan^{-1}(\tan(3\pi/4))$.
    [NCERT; HOTS; Delhi 2011]

✤ Long Answer I / Long Answer II Type

14. Prove that $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$
15. Prove the following: $(9\pi/8) - (9/4)\sin^{-1}(1/3) = \sin^{-1}((2\sqrt{2})/3)$

16. Show that $2\tan^{-1}[\tan(\alpha/2) - \tan((\pi/4) - \beta/2)]$
$= \tan^{-1}[\sin\alpha\cos\beta / (\cos\alpha + \sin\beta)]$ **[NCERT Exemplar]**
17. Write the following function in the simplest form:
$\sin^{-1}[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}]$.
[DoE; HOTS]
18. Solve the following for x:
$\cos^{-1}((x^2-1)/(x^2+1)) + \tan^{-1}((2x)/(x^2-1)) = (2\pi/3)$
19. Prove that $\sin^{-1}(63/65) = \sin^{-1}(5/13) + \cos^{-1}(3/5)$.
[Foreign 2012]
20. Find the value of the following:
$\tan^{-1}(2x/1+x^2) + \cos^{-1}((1-y^2)/1+y^2)$ | | $b < 1, y > 0$ and $xy < 1$.
[NCERT; D.o.E; Delhi 2013]
21. If $\cos^{-1}(x/2) + \cos^{-1}(y/3) = 0$, then prove that
$9x^2 - 12xy\cos 0 + 4y^2 = 36\sin^2 0$. **[HOTS]**
22. Evaluate $\cot(\sqrt{(1+x^2} + x))$.
23. Solve the following equation: $\sin^{-1}(1-x) - 2\sin^{-1}x = \pi/2$
[NCERT; Panchkula 2015]
24. Evaluate $\tan[2\tan^{-1}(1/5) + \pi/4]$ **[Ajmer 2015]**
25. Solve for x, $\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$
[NCERT; Delhi 2019, 2013(C)]
26. Solve for x: $\tan^{-1}(\sqrt{(1+x^2)} - \sqrt{1-x^2})/(\sqrt{(1+x^2)} + \sqrt{1-x^2}) = \beta$.
27. Find the solution of the equation
$\tan^{-1}x - \cot^{-1}x = \tan^{-1}(1/\sqrt{3})$ **[NCERT Exemplar]**
28. If $\tan^{-1}(1/(1+1.2)) + \tan^{-1}(1/(1+2.3)) + \ldots +$
$\tan^{-1}(1/(1+n.(n+1))) = \tan^{-1}0$, then find the value of 0.
[D.o.E; Forrige 2015]
29. Find the principal value of $\tan^{-1}(\tan 5\pi/6)$ **[D.o.E]**
30. Prove that
$\tan^{-1}((\cos x)/(1-\sin x)) - \cot^{-1}(\sqrt{(1+\cos x)/(1-\cos x)}) = \pi/4$,
$x \in (0, \pi/2)$. **[D.o.E]**
31. Evaluate $\tan[1/2\cos^{-1}(3/\sqrt{11})]$ **[D.o.E]**
32. If $\tan^{-1}a + \tan^{-1}b + \tan^{-1}c = \pi$, then prove that $a + b + c = abc$.

ASSESS YOURSELF

1. The equation $\tan^{-1}x - \cot^{-1}x = \tan^{-1}(1/\sqrt{3})$ has solution as _____
2. If $\alpha \leq 2\sin^{-1}x + \cos^{-1}x \leq \beta$, then $\alpha =$ ___, $\beta =$ ___
3. If $\tan^{-1}x = \pi/10$ for some $x \in R$, then the value of $\cot^{-1}x$ is
   (a) $\pi/5$ (b) $2\pi/5$ (c) $3\pi/5$ (d) $4\pi/5$
4. Show that $\sin^{-1}(\sqrt{(a - x)/2a}) = (1/2)\cos^{-1}x/a$.

Write the principal values in Exercises 5 to 8:

5. $\text{cosec}^{-1}(2)$
6. $\cos^{-1}(-\sqrt{3}/2)$
7. $\tan^{-1}(-\sqrt{3})$
8. $\text{Tan}^{-1}(\tan 3\pi/4)$

Write the value in Exercises 9 to 11:

9. $\text{cosec}^{-1}(\sqrt{2}) + \sec^{-1}(\sqrt{2})$
10. $\cos^{-1}(\cos 2\pi/3) + \sin^{-1}(\cos 2\pi/3)$

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